

# Technical Comments

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the Journal of Guidance, Control, and Dynamics are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one Journal page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.

## Comment on “Online Estimation of Allan Variance Parameters”

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IN Ref. 1 the authors state “modern gyros provide angular rate measurements directly, and hence, angular quantization is meaningless.” In fact, any high-precision gyro must employ angular quantization. Rate quantization errors, if assumed to be random, result in unacceptable random walk attitude angle errors with variance increasing proportional to time. Angle quantization, on the other hand, leads to attitude errors bounded in magnitude by one-half quantum.

In a classical example, a caging loop generates a voltage proportional to the input angular rate. The voltage drives a voltage-controlled oscillator generating an output waveform with frequency proportional to the angular rate input. The oscillator output is converted to a train of pulses that drive an up/down counter using appropriate sign logic. At the gyro output sampling rate, the counter output is nondestructively read out, back differenced, and divided by the sampling period to give the rate output. Clearly the pulses represent quanta of angle, not rate.

Such techniques also alleviate the related problem of rate variations at frequencies above the Nyquist frequency. The gyro outputs are actually the average rate over the previous sampling period rather than the instantaneous rate at the sampling instant. The average rate can be discretely integrated without loss of information, whereas the instantaneous rate cannot. (Strictly speaking, both this perfect integration and the one-half quantum bound are achieved only for single-axis rotations. The noncommutativity of rotations introduces second-order errors from both sources during general rotations.)

In error analysis it is important to model properly the quantization. If angle quantization is erroneously modeled as rate quantization the resulting attitude errors will be exaggerated. The simplest correct angle quantization model is a continuous time integrator with angular rate input whose output is quantized, sampled, back differenced, and divided by the sampling period to give the angular rate output.

### Reference

<sup>1</sup>Ford, J. J., and Evans, M. E., “Online Estimation of Allan Variance Parameters,” *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 980–987.

## Reply by the Authors to J. C. Wilcox

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OUR statement “Modern gyros provide angular rate measurements directly, and hence, angular quantization is meaningless” made in the original paper<sup>1</sup> should first be read with the accompanying sentences in the paragraph. The meaning of the sentence would perhaps have been clearer if written “. . . and hence, the term ‘angular quantization’ is misleading.” It should also be noted that this statement is in the nature of an observation and has no impact on the technical contributions in the paper.

The principal concern in this paragraph is that the use of the terminology in the literature can be misleading to a casual reader. Although there may be more than one quantization error source in some devices, the most obvious to a casual reader is the direct quantization of the reported output, regardless of the type of output.

The angular quantization model presented in many publications actually refers to an internal device effect rather than to quantization of a device’s output. In the absence of a separate reference to the quantization of the device’s angular output (which is the case in many publications), a casual reader is apt to draw the mistaken conclusion that these effects are in fact the same.

A second concern is the significance of the contribution of internal angular quantization to the overall device model. Neither our analysis of performance of gyros, nor any reported results for gyro characterizations of which we are aware, showed any significant contribution from internal angular quantization noise. Hence, from a device characterization perspective, inclusion of internal angular quantization appears unnecessary.

### Reference

<sup>1</sup>Ford, J. J., and Evans, M. E., “Online Estimation of Allan Variance Parameters,” *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 980–987.

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